Biosignal Analysis
Biosignal Processing Methods

JH van Bemmel, MA Musen:
Handbook of medical informatics, Springer 1997
Introduction

Fig. 8.1: The four stages of biosignal processing

Types of Signals

Fig. 8.2: Types of biological signals
Types of Signals

Figure 8.3: Impulse series

Analog-to Digital Conversion

<table>
<thead>
<tr>
<th>Signal</th>
<th>Bandwidth (Hz)</th>
<th>Amplitude range</th>
<th>Quantization (bits)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electroencephalogram</td>
<td>0.2-50</td>
<td>600 V</td>
<td>4-6</td>
</tr>
<tr>
<td>Electrooculogram</td>
<td>0.2-15</td>
<td>10 mV</td>
<td>4-6</td>
</tr>
<tr>
<td>Electrocardiogram</td>
<td>0.15-150</td>
<td>10 mV</td>
<td>10-12</td>
</tr>
<tr>
<td>Electromyogram</td>
<td>20-8000</td>
<td>10 mV</td>
<td>4-8</td>
</tr>
<tr>
<td>Blood pressure</td>
<td>0-60</td>
<td>400 mm Hg</td>
<td>8-10</td>
</tr>
<tr>
<td>Spirogram</td>
<td>0-40</td>
<td>10 L</td>
<td>8-10</td>
</tr>
<tr>
<td>Phonocardiogram</td>
<td>5-2000</td>
<td>80 dB</td>
<td>8-10</td>
</tr>
</tbody>
</table>

Table 8.1: Bandwidths, Amplitude Ranges, and Quantization of Some Frequently Used Biosignals.
Analog-to Digital Conversion

08.01 Sampling of Signals: How Often?

Without exception, all biosignals are analog signals. Processing of biosignals by computers therefore requires discretization (i.e., sampling and quantification). This panel explains the sampling process without referring to formulas.

The sampling theorem mathematically phrased by Shannon and Nyquist states that a signal must be sampled at a rate at least twice the rate of the highest-frequency component present in the signal. If we use a sampling rate that is too low, the signal is distorted. If we obey the sampling theorem, all the syntactic information content of the signal is retained. This is illustrated by the following example.

An EEG usually contains statistical, more or less sine wave-shaped fluctuations that may occur at a rate of up to 30 times/second. This can also be expressed by saying that the EEG contains frequencies up to 30 Hz. Higher frequencies may also be present (e.g., from other signal sources) but these are generally not of semantic interest.

The sampling theorem then prescribes that we should sample the EEG at least at 2 x 30 = 60 Hz to keep all signal properties. Table 8.1 gives the frequency bandwidths of interest and the most commonly used sampling rates for some biosignals. For instance, for ECGs (bandwidth, 0.15-150 Hz) a sampling rate above the Shannon frequency (500 Hz) is most often used. If we obey the rules of the sampling theorem it is, in principle, possible to restore the original analog signal by digital-to-analog conversion.

Analog-to Digital Conversion

08.02 Sampling of Signals: How Accurate?

When sampling a signal, we use an analog-to-digital converter (A-D converter or ADC). Samples are taken at a rate at least twice the rate of the highest-frequency component contained in the signal (i.e., the mixture of signal plus noise, unless the noise has been filtered out beforehand), and the samples are quantitated and expressed as numbers. The latter is always done with a limited accuracy and may, in principle, add so-called quantization noise to the sampled signal. This quantization noise should generally not exceed the noise that is already present in the signal, or, as expressed in more general terms, discretization by the ADC should not increase the information entropy (see Chapter 2); syntactic and semantic signal properties should be left intact.

The degree of quantization can be expressed as the number of quantization steps for the range of possible amplitude values. If the signal amplitude spans a range of A volts (e.g., from -A/2 to +A/2) and the quantization step is Δq, then the number of quantization steps is n = A/Δq.

In practice, let n be a power of 2: n = 2^m, so that the quantization of the ADC can be expressed in m bits. For instance, an ADC with an accuracy of 10 bits can discern 2^10 = 1024 different amplitude levels, resulting in a resolution of about 0.1%, expressed as a percentage of the signal range A. An ADC that delivers samples with 8-bit accuracy (2^8 = 256 steps) is called an 8-bit ADC. A 1-bit ADC only determines the sign of the signal (or whether it is larger or lower than some threshold).

For most biosignals a 6- to 12-bit ADC is sufficient; a 12-bit ADC implies a resolution of 1/4096 (less than 0.025%), related to a signal-to-noise ratio which is far superior to that attainable with most signal transducers.
Analog-to Digital Conversion

![Graph showing effect of sampling frequencies](image)

**Figure 8.4:** Effect of sampling frequencies

Application Areas of Biosignal Analysis

![Diagram showing four different situations in biosignal processing](image)

**Figure 8.5:** Four different situations in biosignal processing: output signal, evoked signal, provocative test, process modelling.
Biosignal Processing Methods

Signal-Amplitude Properties

Figure 25.1: Amplitude distribution functions (density distribution function (ddf))
Signal-Amplitude Properties

Figure 25.2: Examples of 2D amplitude distributions

Signal-Amplitude Properties

Figure 25.3: Vectorcardiogram
Signal-Amplitude Properties

Fig. 25.4: Two-dimensional amplitude ddfs for EEG amplitudes and RR intervals

26.6 One-Dimensional Distributions, Means, and Variances

In many cases we want to follow how signals tend to change as a function of time in order to detect trends in the underlying biological processes. In that case we can examine parameters for short observation periods $T$, on which the period $T$ may have some overlap. During such an observation period we consider the signal for study, and determine for each period $T$ parameter values that describe the signal. In this manner we derive parameter values from amplitude ddfs, which may or may not have a stable mean, which may be normal or variable, and which may vary in many different ways, if the signal can be described as

$$\text{signal for period } T = \{e_1, e_2, \ldots, e_T\}$$

where $T$ is the observation period.

The sampled signal can be expressed as

$$x[n] = [e_1, e_2, \ldots, e_T]$$

The interval $T =$ $T/N$ is called the sampling interval, and $f = 1/T$ is the sampling frequency. The amplitude ddf of $x[n]$ can be expressed as $E(x)$, in which two time information or sample number information is kept. The mean of a statistical variable $x$, for which the ddf $E(x)$ is given, may be written as the expression $E(x)$, which is also called the first-order moment of the ddf.

Another way of writing the mean value of the variable $x$ is

$$E[x] = \int x f(x) \, dx$$

which is a good approximation of $E(x)$ for large $N$. Thus, for the following approximation of $E(x)$ is obtained. For arbitrary signals, the expression holds for all observation periods $T$, for stationary signals may vary. In a similar way, it is possible to write for the variance of the signal the second-order moment of the ddf.

$$E[(x - E(x))^2] = \int (x - E(x))^2 f(x) \, dx$$

For a signal with a mean value zero $E(x) = 0$, the variance becomes

$$E(x^2) = \int x^2 f(x) \, dx$$

The exponent $q$, is the square root of the variance. The signal-to-noise ratio (SNR) of a signal is the ratio of the root mean square value of the signal to the noise (which is calculated as the mean squared value of the noise).

$$\text{SNR} = S/N = \sqrt{E(x^2)}/\sqrt{E(x^2)}$$

$$E(x^2) = E(x^2)$$
Frequency Spectra and Filtering

Figure 25.5: Examples of three biological signals with their frequency spectra
Frequency Spectra and Filtering

Figure 25.6: Frequency components.

25.03 Wavelet Analysis

In Fourier analysis, a signal is thought to be composed of sines and cosines. By using the Fourier transform, a signal can be decomposed into these basic functions. The samples of the transformed signal (Fourier coefficients) represent the contribution of sine and cosine functions at different frequencies. A disadvantage of Fourier analysis is that it is difficult to compose a signal that is limited in time, by using functions that, by definition, stretch out into infinite time. It is therefore difficult for a Fourier function to approximate sharp changes in a signal. For example, a very simple and time-limited signal, a spike, is decomposed by Fourier transformation into an infinite number of sines and cosines (see also Fig 25.6a for the decomposition of a block signal).

A way to tackle this problem is through wavelet analysis. It uses the same principle as Fourier analysis, namely, that signals are composed of basic functions, called wavelets. The most important difference between these wavelets and the sines and cosines used in Fourier analysis is that wavelets are limited in time. The procedure for wavelet analysis is to choose a suitable wavelet prototype function (also called mother wavelet or analyzing wavelet) that meets certain constraints. All composing functions are derived by stretching or scaling the mother wavelet both in time and in amplitude. Using a wavelet transform, the signal is decomposed into these scaled versions of the mother wavelet. In fact, the composing cosines used in Fourier analysis can also be seen as stretched, scaled, and shifted versions of a mother-cosine. In Fourier analysis, the composing functions are infinite in the time domain because they represent exactly one frequency. In wavelet analysis, the composing wavelets have a limited extent both in the time domain and in the frequency domain, where contributions from frequencies outside a certain area are negligible.

The most important result of the wavelet transform is the location of the composing wavelets in time. Sharp, time-limited signal parts will be represented by wavelets that are scaled-down in duration. As in Fourier analysis, the contribution of the composing wavelets to the signal provides information about the temporal properties of the signal on different time scales. Additionally, the locations of the composing wavelets provide information about the position of a specific signal property. Figure 25.7 shows two examples of a mother wavelet, both from the well-known Coiflet wavelet family.
Frequency Spectra and Filtering

Figure 25.7: Two examples of a mother wavelet.

Figure 25.8: Frequency spectra of an ECG.
Figure 25.9: Schematic representation of filters.

Figure 25.10: ECG and its band-pass filtered version.
Frequency Spectra and Filtering

15.01 Relationships between True Positive (TP), True Negative (TN), False Positive (FP), and False Negative (FN)

<table>
<thead>
<tr>
<th>Truth</th>
<th>TP</th>
<th>FN</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td></td>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>-</td>
<td>FP</td>
<td>TN</td>
<td></td>
</tr>
</tbody>
</table>

Table 15.1. Relationships between True Positive (TP), True Negative (TN), False Positive (FP), and False Negative (FN).

15.02 Illustration of Sensitivity, Specificity, and Predictive Value

<table>
<thead>
<tr>
<th>Truth</th>
<th>a</th>
<th>b</th>
<th>a + b</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>c</td>
<td>d</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a + c</td>
<td>b + d</td>
<td>a + b + c + d</td>
</tr>
</tbody>
</table>

Table 15.2. Illustration of Sensitivity, Specificity, and Predictive Value (see text).

\[
\text{sensitivity} = \frac{a}{a+b} \quad \text{ppv} = \frac{a}{a+c} \\
\text{specificity} = \frac{d}{c+d} \quad \text{npv} = \frac{d}{b+d}
\]

Frequency Spectra and Filtering

Figure 15.6: Distributions of systolic blood pressure of hypertensive and nonhypertensive people: a) population survey, b) primary care, c) cardiac clinic
Frequency Spectra and Filtering

**Figure 15.7**: Distributions of the primary population of Fig. 15.6.

Frequency Spectra and Filtering

**Figure 15.8**: ROC curves of the population of Fig. 15.7
TN (100-FP) vs. FN (100-TP).
Signal-to-Noise Ratio

Figure 25.11: Coherent averaging in an ECG recording.

Coherent Averaging

In coherent averaging we compute the sum of, say, \( K \) waveforms \( s_0 \), which are extracted after detection from a noisy signal \( x_0(t) = s_0(t) + n_0(t) \). The original signal variance is \( S_0 = \sigma_{s_0}^2 \) and the noise variance is \( N_0 = \sigma_{n_0}^2 \), so that the SNR is:

\[
\text{SNR}_0 = \frac{S_0}{N_0}.
\]

The sum of the \( K \) signal waveforms \( s_0 \) will result in a waveform \( s_1 \) which is \( K \) times as large as the original waveform, that is, \( s_1 = Ks_0 \). The resulting signal dispersion is also \( K \) times as large: \( \sigma_{s_1} = K\sigma_{s_0} \). The variance of \( s_1 \) is then:

\[
S_1 = \sigma_{s_1}^2 = K^2\sigma_{s_0}^2.
\]

We assume that the noise has a normal distribution. The \( K \) noisy waveforms \( n_0 \) are also summed to a new noisy signal \( n_1 \). It can be proved that the variance of \( n_1 \) is \( K \) times as large as the variance of \( n_0 \) so that \( n_1 = Kn_0 \). The SNR after summation is then:

\[
\text{SNR}_1 = \frac{S_1}{N_1} = \frac{K^2\sigma_{s_0}^2}{K^2\sigma_{n_0}^2} = \frac{\sigma_{s_0}^2}{\sigma_{n_0}^2} = K^{\frac{2}{2}} \frac{S_0}{N_0} \frac{1}{K} = K \frac{S_0}{N_0}.
\]

This implies that the SNR has improved linearly with the number of summed waveforms.
Signal Detection

### 25.01 Four Different Detection Situations for the Decision D that an event S is Present

<table>
<thead>
<tr>
<th>Situation</th>
<th>Description</th>
<th>S</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>TP</td>
<td>The event is present AND is correctly detected</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>FP</td>
<td>The event is not present AND is incorrectly detected</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>TN</td>
<td>The event is not present AND is correctly not detected</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>FN</td>
<td>The event is present AND is incorrectly not detected</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 25.1: Four Different Detection Situations for the Decision D that an event S is Present.

*TP, true positive; TN, true negative; FP, false positive; FN, false negative.*

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**Figure 25.12:** Detection and estimation may reinforce each other.
Signal Detection

Figure 25.12: An artificial signal of amplitude-modulated impulses.